Hampshire Services
HIAS SCHOOL IMPROVEMENT

MATHEMATICS ADVISORY CENTRE

# Progression in Calculation
# Addition and Subtraction

Hampshire County Council

www.hants.gov.uk

Version 1 - October 2016

## How to use this document

This document is designed to give ideas about how to use concrete apparatus and images to support children's conceptual understanding.

So often children are able to follow calculation processes and get the answer right in many cases without fully understanding how and why the method works. This document is designed to support teachers to do that so that children are able to reason effectively and apply maths that they have learnt to other methods and new concepts as they have that deep understanding required at Mastery. Not all National Curriculum objectives are listed but just some to give a flavour of how the images and apparatus can be used to develop children's conceptual understanding of different mathematical ideas. Teachers should read previous year groups to ensure children have consolidated these ideas before moving on. For some year groups the same images and concrete resources will be used but maybe with larger numbers and with a greater level of reasoning so they have therefore not been repeated.

For more guidance use the Resource Tool https://www.ncetm.org.uk/resources/41211 on the NCETM website where exemplification material and activity ideas can be found using the following tabs of EXEMPLIFICATION and ACTIVITIES. The SUBJECT KNOWLEDGE Audit tool will also help to give an insight to the expectations under each NC objective. There is also a tab for VIDEOS under each domain where you can see teacher using concrete resources to develop childrens' conceptual understanding and reasoning.

# The EYFS Framework

Mathematics involves providing children with opportunities to develop and improve their skills in counting, understanding and using numbers, calculating simple addition and subtraction problems; and to describe shapes, spaces, and measures

**Early Learning Goals**

Mathematics Numbers: children count reliably with numbers from 1 to 20, place them in order and say which number is one more or one less than a given number. Using quantities and objects, they add and subtract two single-digit numbers and count on or back to find the answer. They solve problems, including doubling, halving and sharing.

Shape, space and measures: children use everyday language to talk about size, weight, capacity, position, distance, time and money to compare quantities and objects and to solve problems. They recognise, create and describe patterns. They explore characteristics of everyday objects and shapes and use mathematical language to describe them.

Additional Guidance can be found in Development Matters:

https://www.early-education.org.uk/development-matters

# Early Maths Foundation Stage to early KS1

ELG: Children count reliably with numbers from one to 20, place them in order and say which number is one more or one less than a given number. Using quantities and objects, they add and subtract two single-digit numbers and count on or back to find the answer. They solve problems, including doubling, halving and sharing.

## Counting

- Children count reliably with numbers from one to 20, place them in order and say which number is one more or one less than a given number.

### Oral counting

Oral counting in 1s forwards and backwards to 10 then 20 **starting at zero. 0,1,2,3 etc**

Progress to **starting at any number** and counting in 1s. 5, 6, 7 (important if children are able to count on later.

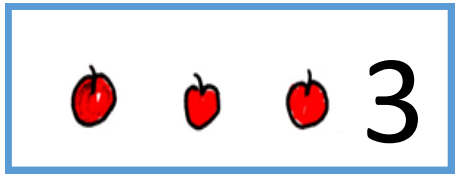
Oral counting- Saying **teen and ty** numbers correctly.

e.g. 13- thirteen, 30- thirty.

40-60 months
Selects the correct numeral to represent 1 to 5, then 1 to 10 objects.

### Object counting

**Counting all- 1:1 principle (1:1 correspondence)**




**One, two, three**

**Counting objects up to 10 then 20.**

Children need to understand that number labels (words) match objects as they count them.

## Place Value and Number System

- Using quantities and objects, they add and subtract two single-digit numbers and count on or back to find the answer

### Subitising

Children should start to recognise small amounts without counting, especially when presented with familiar arrangements e.g. numicon and dice.

### Number conservation

Children should have opportunities to explore groups of objects and note that when some are moved there is still the same quantity there (unless any are removed or added).

How many counters?

How many now?

# The learning environment

# A number rich environment

### Multi-representation of numbers to 20

Activities to promote multi-representation through play/exploration.

**In your writing area– have you got prompts to encourage children to write their numbers and mark make in maths?**

40-60 months

- Counts up to three or four objects by saying one number name for each item.
- Counts actions or objects which cannot be moved.
- Counts objects to 10, and beginning to count beyond 10.
- Counts an irregular arrangement of up to ten objects.
- Counts out up to six objects from a larger group.
- Counts an irregular arrangement of up to ten objects.

Children need opportunities to make links with number labels (figures),

object counting and ordering numbers.

# Place Value and Number System

### Ordering numbers

Ordering a set of consecutive numbers e.g.

| | | | | |
|---|---|---|---|---|
| 5 | 6 | 7 | 8 | 9 |

Ordering a set of random numbers e.g.

| | | | | |
|---|---|---|---|---|
| 2 | 5 | 6 | 7 | 9 |
| 6 | 19 | 11 | 5 | 15 |

### Using comparative language to describe group size

Which group has more/fewer?

Which has most/least?

40-60 months
Uses the language of 'more' and 'fewer' to compare two sets of objects.

There are fewer apples/less apples than oranges.

There are more oranges than apples.

There are most oranges.

# Addition

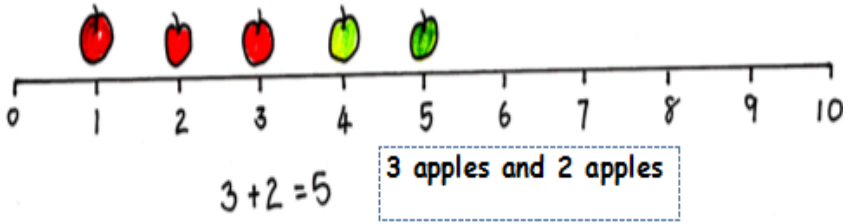
## Aggregation– combining groups

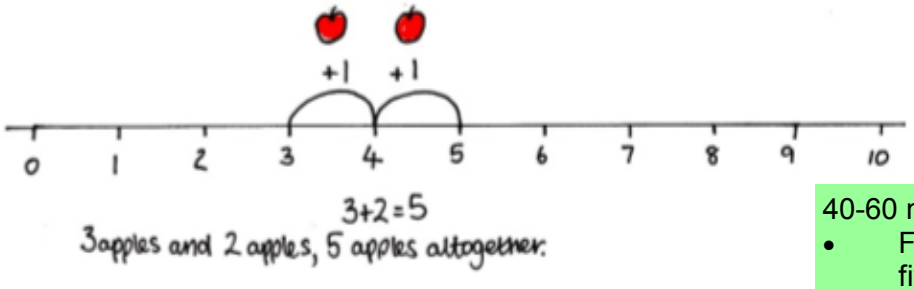
Counting all, 1, 2, 3, 4, 5 There are 5 apples

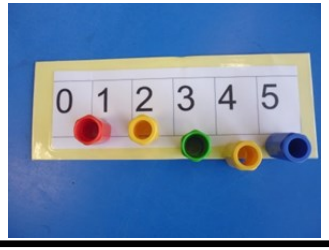
## Structured Number Lines

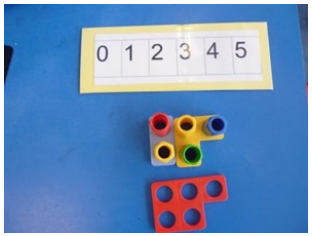
Counting on from first number **2,** 3, 4, 5 There are 5 apples

Counting on from the greatest number. **3,** 4, 5

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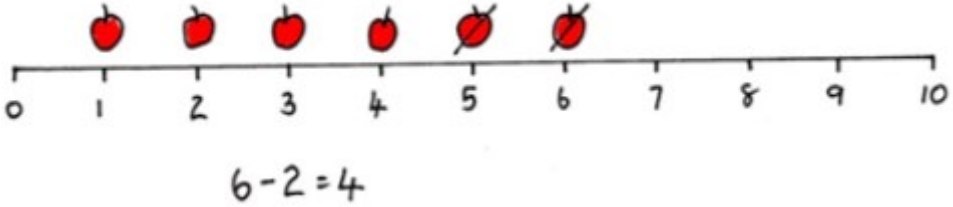
# Subtraction

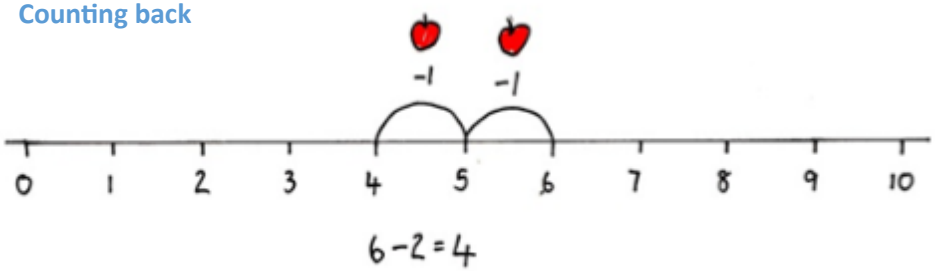
## Taking away– removing objects from a group

**I have 6 apples. I eat 2 apples. How many are left?**

**Count out 6, take away 2, count how many are left?**




## Counting back




40-60 months

- Finds one more or one less from a group of up to five objects, then ten objects.
- In practical activities and discussion, beginning to use the vocabulary involved in adding and subtracting.

• In practical activities and discussion, beginning to use the vocabulary involved in adding and subtracting.

# Exploring relationships (inverse)

**Bar Modelling**

**Part, part, whole models**

40– 60 months

- In practical activities and discussion, beginning to use the vocabulary involved in adding and subtracting.

***What is the same? What is different?***

*Mary had 7 letters in her bag and she posted 3. How many did she have left?*

*7—3 = ?*

*Mary had 7 letters in her bag and after she posted some, she had 4 left. How many did she post?*

*7—? = 4*

*Mary had some letters and after posting 3, she had 4 left. How many did she start with?*

*? - 3 = 4*

**Act out problems in different ways**

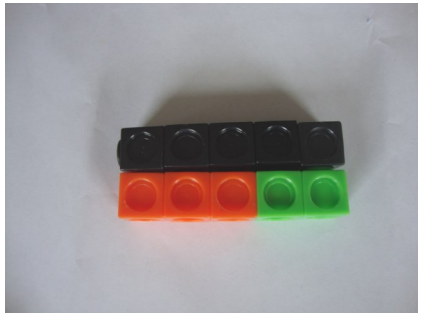
The whole is 5.

3 is a part, 2 is a part of the whole.

If you remove 1 part, the other is left. E.g. 5 - 3 = 2 or 5 - 2 = 3

If you put the parts together, you get the whole.

3 + 2 = 5 2 + 3 = 5 These are commutative .



Year 1

- Read, write and interpret mathematical statements involving addition (+), subtraction (-) and equals (=) signs
- Solve one-step problems that involve addition and subtraction, using concrete objects and pictorial representations, and missing number problems such as 7 = ? - 9

# Key Stage 1

The principal focus of mathematics teaching in key stage 1 is to ensure that pupils develop confidence and mental fluency with whole numbers, counting and place value. This should involve working with numerals, words and the four operations, including with practical resources [for example, concrete objects and measuring tools].

At this stage, pupils should develop their ability to recognise, describe, draw, compare and sort different shapes and use the related vocabulary. Teaching should also involve using a range of measures to describe and compare different quantities such as length, mass, capacity/volume, time and money.

By the end of year 2, pupils should know the number bonds to 20 and be precise in using and understanding place value. An emphasis on practice at this early stage will aid fluency.

Pupils should read and spell mathematical vocabulary, at a level consistent with their increasing word reading and spelling knowledge at key stage 1.

**National Curriculum 2014**

# Key stage 1

## Counting

Concentrate on the tricky areas e.g. bridging through 100.

97, 98, 99, 100, 101, 102

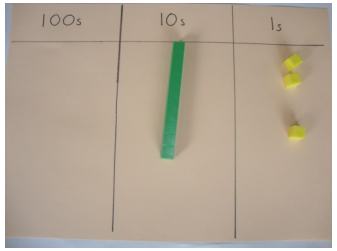
Remember to count backwards as frequently as you count on!

Year 1
Count to and across 100, forwards and backwards, beginning with 0 or 1 or from any given number.
Count, read and write numbers to 100 in numerals.
Given a number, identify one more, one less.

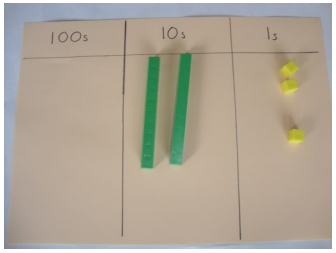
Year 2
Count in tens from any number (forwards and backwards)

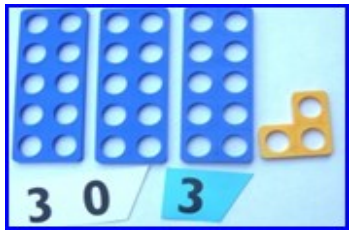
What has changed? Stayed the same?

13, 23, 33




+ 10 →




## Place Value and Number System

### Ordering numbers

Which numbers are covered?

Give me a number between… and… .
How do you know?

Order consecutive to 100. Use number line to support.

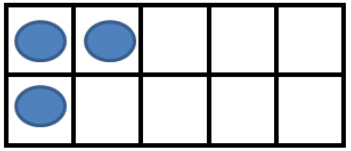
Year 1
- Read and write numerals from 1 to 20 in numerals and words.
- Identify and represent numbers using objects and pictorial representations including the number line, and use the language of: equal to, more than, less than (fewer), most, least

### Number formation

Stencils, dot to dot, tracing, writing in sand, making numbers from modelling dough, using different pens.

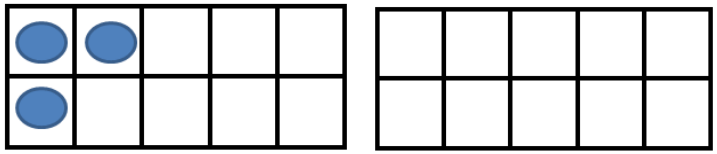
Year 1
Count read and write numbers to 100 in numerals.

# Addition



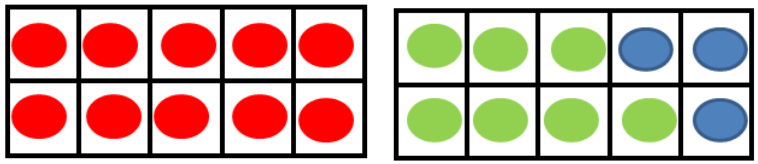
$\boxed{3} + \boxed{\phantom{0}} = 10$

How many to make 10? What about 20?



**Partitioning numbers in different ways**

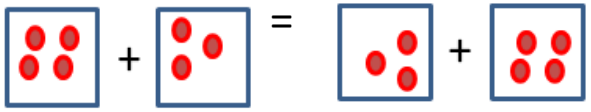
$20 = \boxed{10} + \boxed{7} + \boxed{\phantom{0}}$



Lucy has 20 marbles in her bag. 10 were red, 7 were green and the rest were blue. How many

**Commutativity**

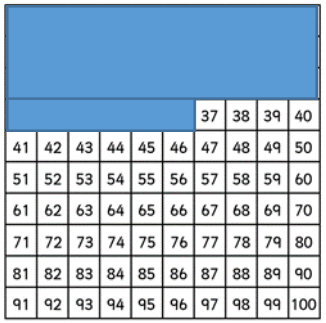
4 + 3 = 3 + 4



# Subtraction

Year 2
- derive and use related facts up to 100.

$36 + \boxed{\phantom{00}} = 100$



| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | | 37 | 38 | 39 | 40 |
| 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 |
| 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 |
| 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 |
| 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 |
| 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 |
| 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 |

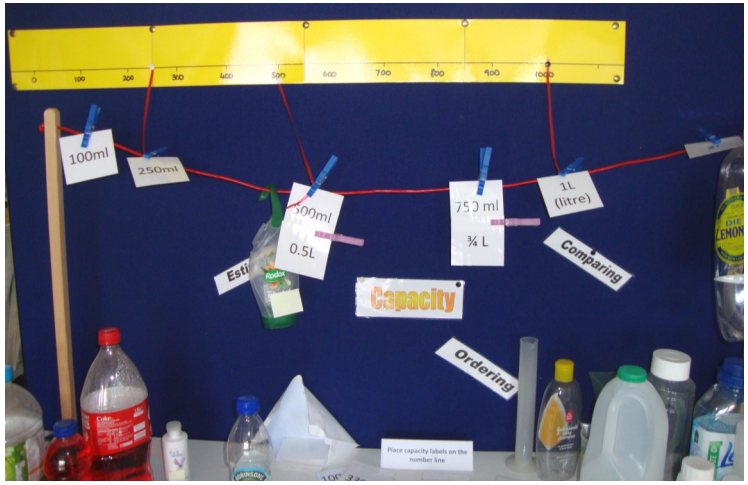
Year 2
- Solve problems with addition and subtraction: using concrete objects and pictorial representations, including those involving numbers, quantities and measures; apply their increasing knowledge of mental and written methods.

Year 1
- Solve one step problems that involve addition and subtraction, using concrete objects, pictorial representations, and missing number problems.

Jo poured 4 cups of juice from the jug. How many cups were left in the jug?

Children need to use number lines in different orientations e.g. a vertical number line supports work with capacity and Statistics

# The learning environment




A number rich environment

Display numbers in words, numerals and using Multi-representation.

Make link to other areas e.g. measures and money.

# Place Value and Number System

## Ordering numbers

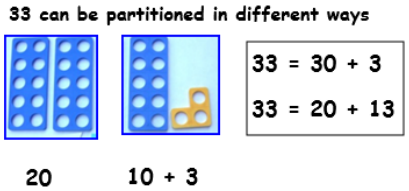
Order consecutive and random numbers to 100.

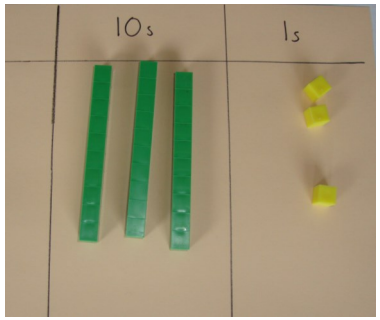
| | | | | |
|---|---|---|---|---|
| 46 | 63 | 72 | 85 | 100 |

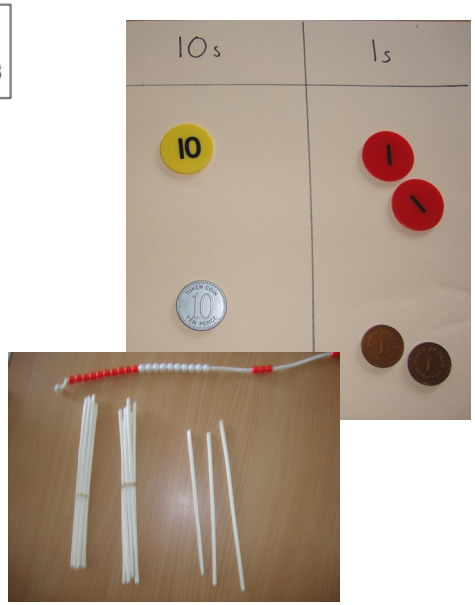
Year 2
Compare and order numbers from 0 up to 100; use the < > and = signs.

## Concept of place value tens and ones

Year 2
Recognise the place value of each digit in a 2 digit number (tens and ones)



# Addition

### Using known facts

Secure bonds to ten

Bonds to 20

Bonds for any number to ten

Then use these for bonds within 20

Year 1

- Represent and use number bonds and related subtraction facts within 20
- Add and subtract one-digit and two-digit numbers to 20, including zero
- Read and write mathematical statements involving addition and subtraction and equals signs.

### Variation

If I know 2 + 3 = 5,

How could this help you with 2 + 4?

What is the same? What is different?

Show me

Vary the way in which you present the practice. Use images and practical apparatus combined with symbols.

□ + □ = 5

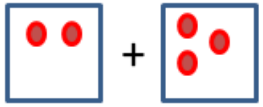
□ + 4 = 5

□ + 3 = 5

How many ways can you make

Continue the pattern. What is the same? What is different?

What other ways can you arrange your counters to make your calculation equal 5?

5 = 

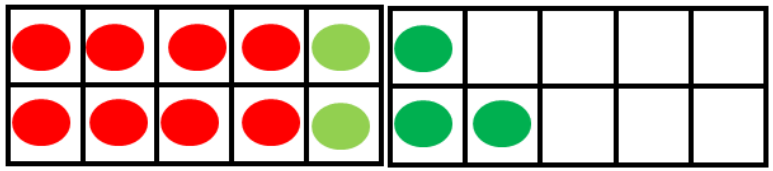
Year 2

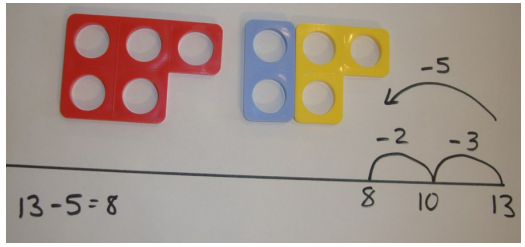
- Recall and use addition and subtraction facts to 20 fluently and derive and use related facts up to 100.

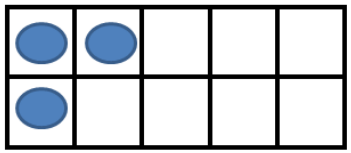
# Subtraction

### Using known facts

13—5 = 13—3—2





10 - □ = 3

20 = □ - □

What subtraction facts to 20 can you show using a range of apparatus ?

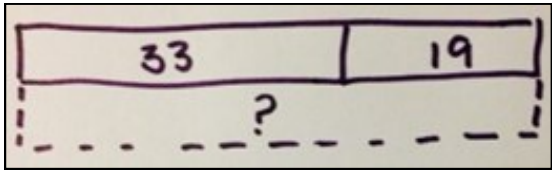
Record as a number sentence.

| | | |
|---|---|---|
| dogs | 𝍸 \|\| | 7 |
| Cats | 𝍸 𝍸 𝍸 \|\|\|\| | 19 |
| Rabbits | 𝍸 𝍸 \|\| | 12 |
| Snake | \| | 1 |
| Hamster | \|\|\| | 3 |
| birds | \|\| | 2 |

# Addition

Unstructured number lines

Jottings to support mental methods  e.g. number line and bar model



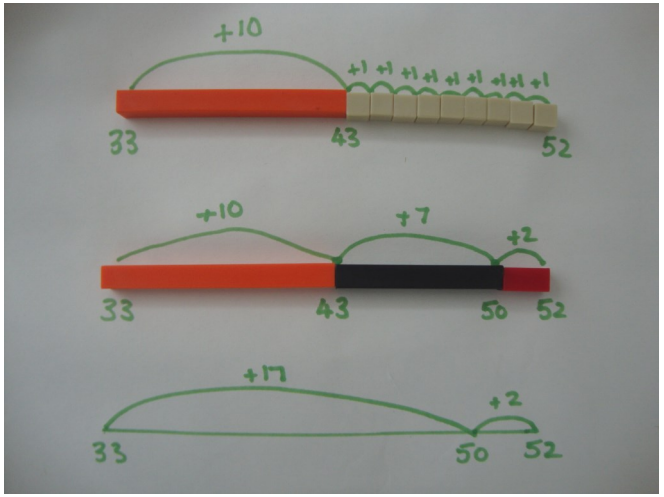

Year 2

- *NON STATUTORY*

*Pupils extend their understanding of the language of addition and subtraction to include sum and difference.*

There are 33 children in the playground, 19 more come out to play. How many children are now in the playground?

33 + 19 = ?




33 + 19

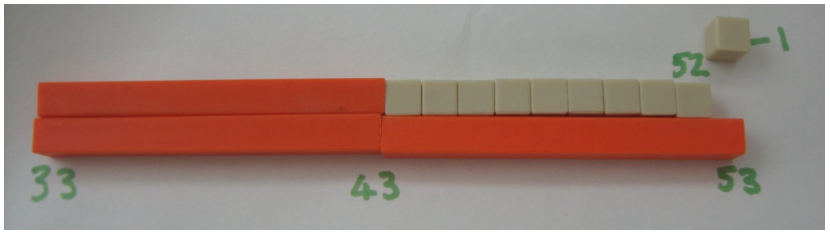
33 + 10 + 7 + 2

33 + 17 + 2

Encourage use of number bonds

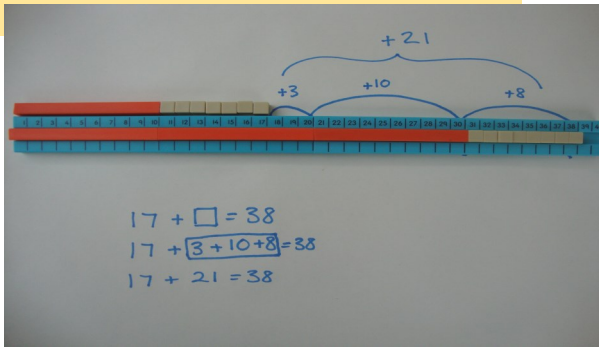
33 + 20 –1

**Round and adjust– adding near tens**



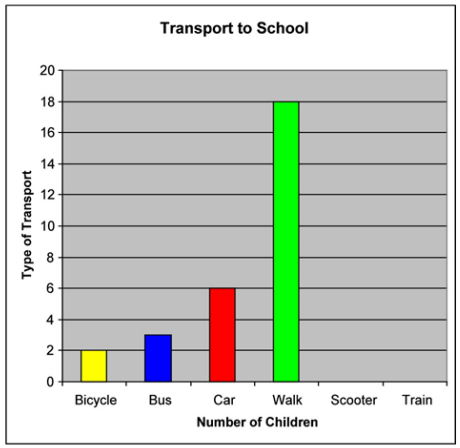

# Subtraction

Finding the difference and counting up to subtract (see also exploring relationships)




I have 36 DVDs, my friend has 17. How many **more** DVDs do I have than my friend?

36 = 17 + ?  or  17 + ? = 36




How many more children walked to school than travelled by car?

Use a variety of contexts for children to practice their addition and subtraction skills.

# Addition

**Jottings to support mental methods e.g. using place value - partitioning**

Once children can count on/back in tens it is easier if they just partition one number as quickly as possible. This prevent issues with subtraction where partitioning both numbers does not work with bridging.

**Progression**

**36 + 40 (adding only tens to any number)**

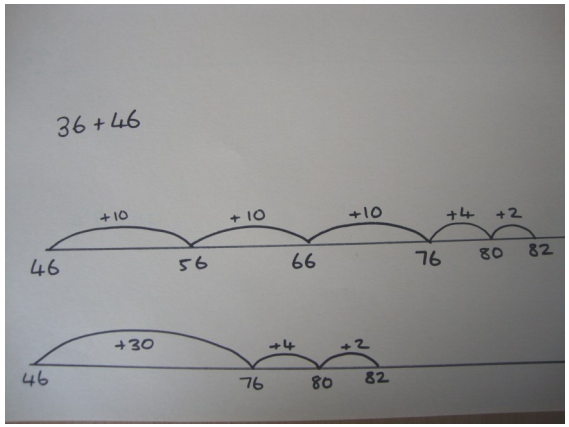
**36 + 43 (adding tens and ones with no bridging)**

**36 + 46 (adding tens and ones with bridging)**

36 + 46

36 + 40 (**36**, 46, 56, 66, 76) add tens

76 + 6 or 76 + 4 + 2 (using number bond knowledge)




Year 2
Add and subtract numbers using concrete objects, pictorial representations, and mentally, including:

- A 2 digit number and ones
- A 2 digit numbers and tens
- Two 2 digit numbers.
- Adding 3 one digit numbers.

# Subtraction

**Jottings to support mental methods e.g. using place value - partitioning**

**56 - 20 (subtracting only tens to any number)**

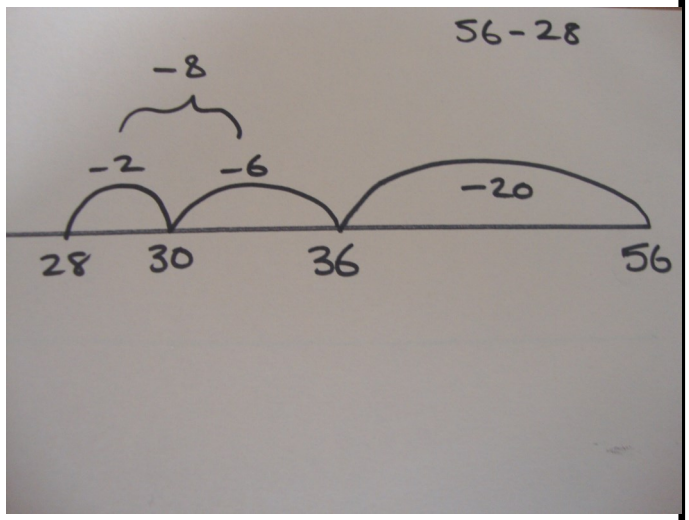
**56 - 23 (subtracting tens and ones with no bridging)**

**56 + 28 (subtracting tens and ones with bridging)**

**56 - 28**

**56,** 46, 36 (-20)

36—8 or 36—6 –2



# Lower Key Stage 2

The principal focus of mathematics teaching in lower key stage 2 is to ensure that pupils become increasingly fluent with whole numbers and the four operations, including number facts and the concept of place value. This should ensure that pupils develop efficient written and mental methods and perform calculations accurately with increasingly large whole numbers.

At this stage, pupils should develop their ability to solve a range of problems, including with simple fractions and decimal place value. Teaching should also ensure that pupils draw with increasing accuracy and develop mathematical reasoning so they can analyse shapes and their properties, and confidently describe the relationships between them. It should ensure that they can use measuring instruments with accuracy and make connections between measure and number.

By the end of year 4, pupils should have memorised their multiplication tables up to and including the 12 multiplication table and show precision and fluency in their work.
Pupils should read and spell mathematical vocabulary correctly and confidently, using their growing word reading knowledge and their knowledge of spelling.

**National Curriculum 2014**

# Recognising place value

**ANY BANK PLC**

Account Payee 20-10-25

Pay _____ Date _____

_____

_____ A N OTHER

| Cheque No. | Branch Sort Code | Account No. |
|---|---|---|
| 001234 | 20-10-25 | 4411223331 |

Lottery win! Write the winning cheques.

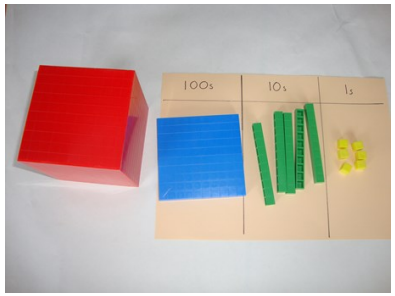
Correct the mistake– 1 person has been paid £100 too much/£1000 too little etc!

Year 3

- *recognise the place value of each digit in 3 -digit number (hundreds, tens, and ones)*
- *find 10 and 100 more or less than a given number*
- *count backwards through zero to include negative numbers*
- read and write numbers to at least 1000 in numerals and in words
- read Roman numerals to 100 (I to C) and understand how, over time, the numeral system changed to include the concept of zero and place value.

Use the pattern to complete the missing values.

| | | | | |
|---|---|---|---|---|
| I | 1 | | XXI | 21 |
| II | 2 | | XXII | 22 |
| III | 3 | | … | 23 |
| IV | 4 | | XXIV | 24 |
| V | 5 | | XXV | 25 |
| VI | 6 | | XXVI | 26 |
| VII | 7 | | XXVII | 27 |
| VIII | 8 | | XXVIII | 28 |
| … | 9 | | XXIX | 29 |
| X | 10 | | XXX | 30 |
| … | 11 | | XXXI | 31 |
| XII | 12 | | XXXII | 32 |
| XIII | 13 | | XXXIII | 33 |
| XIV | 14 | | … | 34 |
| XV | 15 | | XXXV | 35 |
| XVI | 16 | | XXXVI | 36 |
| XVII | 17 | | XXXVII | 37 |
| XVIII | 18 | | XXXVIII | 38 |
| XIX | 19 | | XXXIX | 39 |
| XX | 20 | | XL | 40 |



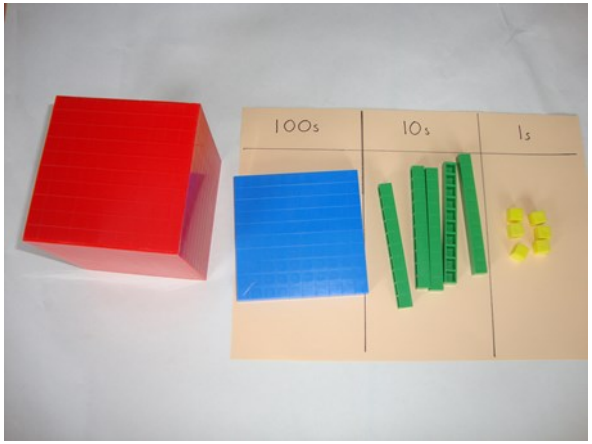

> 1136

Prove it. How do you know? Show me.

Year 4

- *find 1000 more or less than a given number*
- *count backwards through zero to include negative numbers*
- *recognise the place value of each digit in 3 and four-digit number (thousands, hundreds, tens, and ones)*
- read and write numbers to at least 1000 in numerals and in words
- round any number to the nearest 10, 100 or 1000
- read Roman numerals to 100 (I to C) and understand how, over time, the numeral system changed to include the concept of zero and place value.

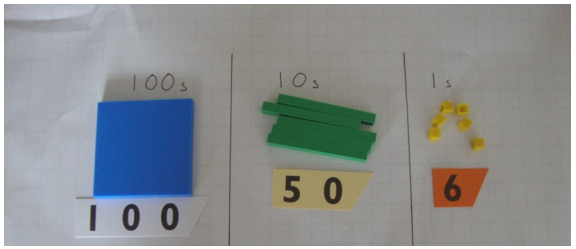
# Recognising place value

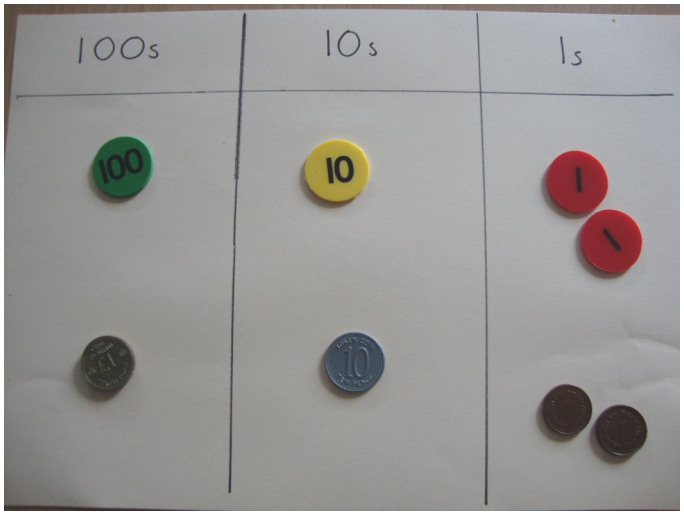



Year 4
- round any number to the nearest 10, 100 or 1000
- order and compare numbers to and **beyond 1000**
- identify, represent and estimate numbers using different representations

What is the same/different about the numbers?

340 – 20 = 360 - □




□□□ + □□= 500

□□□ - □□□ = 320

**Same and different**

Which calculations are the same? How do you know?

230 + 30 210 + 50 290 – 30

245 + 15 247 + 13

Year 3
- Solve problems using missing number problems using number facts, place value and more complex addition and subtraction
- order and compare numbers **to 1000**
- identify, represent and estimate numbers using different representations

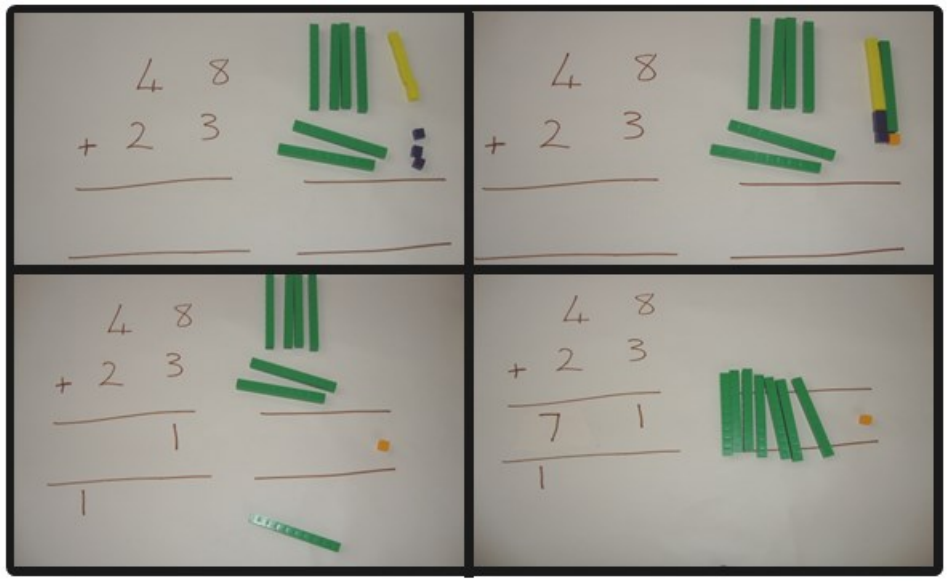
# Addition

## Moving to formal methods

Year 3
- Add and subtract numbers with up to 3 digits, using formal written methods of columnar addition and subtraction.

Year 4
- Add and subtract numbers with up to 4 digits, using formal written methods of columnar addition and subtraction where appropriate

8 + 3 = 11 (exchange for 1 ten and 1 one)

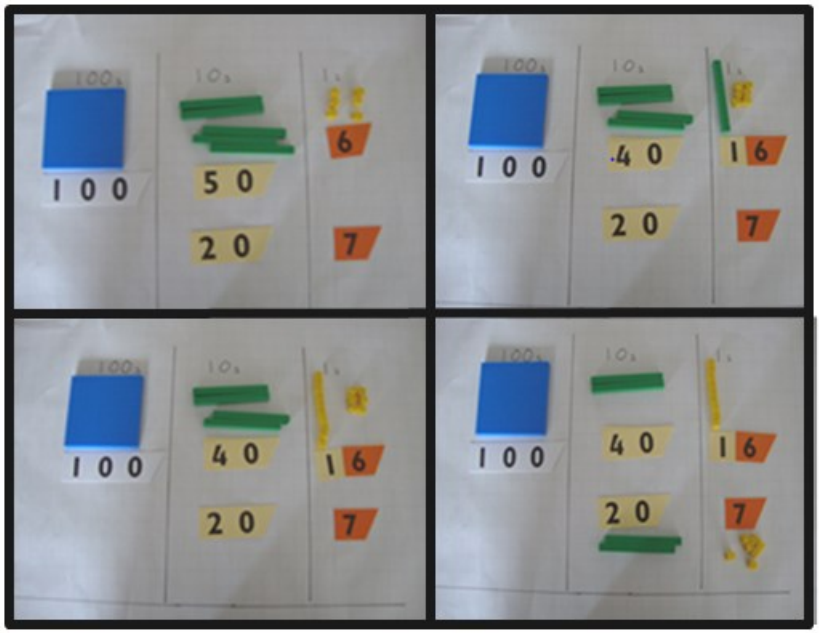



## Key skills progression

Carrying 10s
Carrying 100s/1000s
Carrying in more than 1 column (e.g. 10's and 100's)

# Subtraction

## Moving to formal methods




Model using practical apparatus alongside written methods so children UNDERSTAND what is happening.

## Key skills progression—exchanging or regrouping

No regrouping/exchanging required

Regrouping tens (exchanging from 1s for 10s only)

Regrouping from hundreds only (exchanging from 10s)

Regrouping in more than 1 column (e.g. exchanging both 1s and 10s for 100s)

## Upper Key Stage 2

The principal focus of mathematics teaching in upper key stage 2 is to ensure that pupils extend their understanding of the number system and place value to include larger integers. This should develop the connections that pupils make between multiplication and division with fractions, decimals, percentages and ratio.
At this stage, pupils should develop their ability to solve a wider range of problems, including increasingly complex properties of numbers and arithmetic, and problems demanding efficient written and mental methods of calculation. With this foundation in arithmetic, pupils are introduced to the language of algebra as a means for solving a variety of problems.

Teaching in geometry and measures should consolidate and extend knowledge developed in number. Teaching should also ensure that pupils classify shapes with increasingly complex geometric properties and that they learn the vocabulary they need to describe them.
By the end of year 6, pupils should be fluent in written methods for all four operations, including long multiplication and division, and in working with fractions, decimals and percentages.
Pupils should read, spell and pronounce mathematical vocabulary correctly.

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# Place value, addition and subtraction in context

Year 3
- Add and subtract amounts of money to give change, using both £ and p in practical contexts. (non statutory; including mixed units)

Year 4
Estimate, compare and calculate different measures, including money in pence and pounds
NON STATUTORY- *pupils build on knowledge of place value and decimal notation to record metric measures including money.*

Use < > or = to compare

112p and £1.12

Prove it.

Year 4 Measures
- Measure and calculate the perimeter of a rectilinear figure (including squares) in cm and m

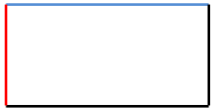
NON STATUTORY- *perimeter can be expressed algebraically as 2(a+b)*

Year 5 Statistics

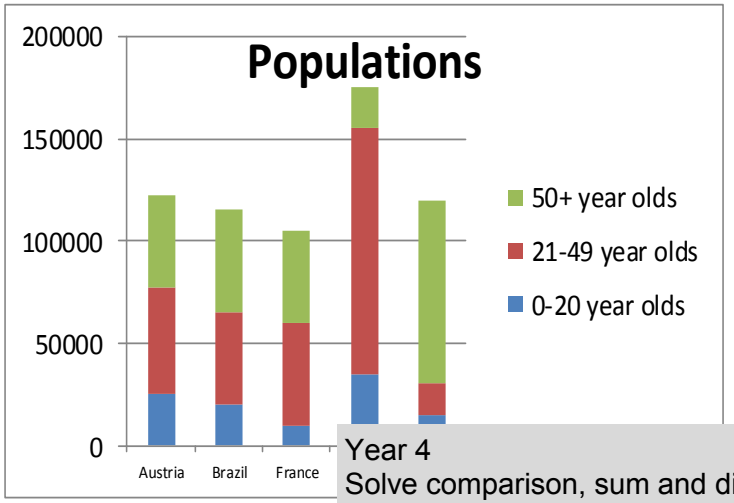
Solve comparison, sum and difference problems using information presented in a line graph.

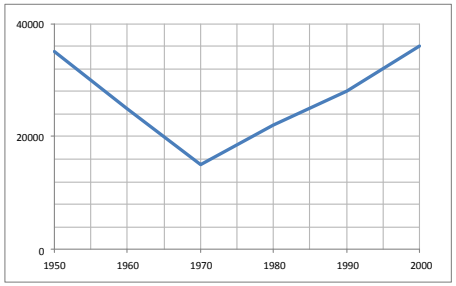
What formula will I need to calculate the perimeter of a rectangle?

2(a + b)



How many rectangles can you draw with a perimeter of…?




How did the population grow between 1970 and 1980?

How many 21-49 year olds live in Austria?

Key

**Blue** = 0-20 year olds

**Red** = 21-49 year olds

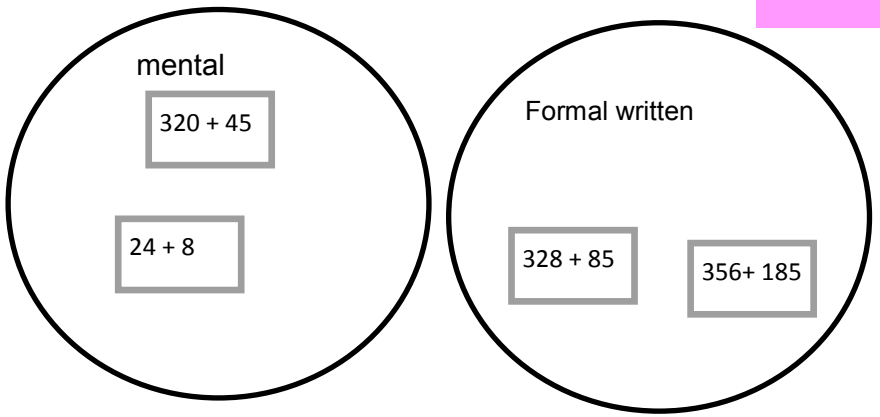
**Green** = 50+ year olds

Year 4
Solve comparison, sum and difference problems using information presented in bar charts, pictograms, tables and other graphs.

# Addition

**Making choices about when formal methods are appropriate**

**Sort these calculations. Explain which methods you would use and why.**




Year 3
Add and subtract numbers mentally, including:
- A three digit number and ones
- A three digit number and tens
- A three digit number and hundreds

Year 4
- Add and subtract numbers with up to 4 digits, using formal written methods of columnar addition and subtraction where appropriate

Year 5 and Year 6

solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why

If there are 2544 people in the stadium on Thursday and 3456 on Friday. How many went to the matches that week?

2500 + 3400 =
2540 + 3450 =
2540 + 3460 =
2500 + 3450 =
2500 + 3460 =

Which will give the largest total?

# Subtraction

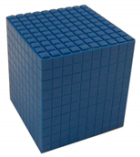
Year 3
- Estimate and check the answer to a calculation and use inverse operations to check answers.

Which of these will give the following approximate answer? 50

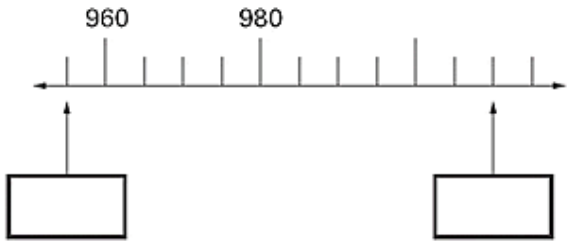
78 – 40
175 – 122
139 – 90
89 - 50

# Place Value and Counting

How many of these would make a million?



Think about how you would record it?




Year 5

- read, write, order and compare numbers to at least 1 00 000 and determine the value of each digit

Read the numbers on the scale.

Which number is between… and …?

Where would 4250 be approximately?

Year 6

read, write, order and compare numbers up to 10 000 000 and determine the value of each digit

Order the planet sizes from small-est to largest

Make a scale model

| Planet | Diameter km |
|---|---|
| Mercury | 4878 |
| Venus | 12104 |
| Earth | 12756 |
| Mars | 6787 |
| Jupiter | 142796 |
| Saturn | 120660 |
| Uranus | 51118 |
| Neptune | 48600 |
| Pluto | 2274 |

Year 5

- count forward or backwards in steps of powers of 10 for any given number up to 1 000 000

2500, 2600, 2700 etc

Will 3850 be in your sequence. Why? Why not?

Create a sequence with 2450 in it. Describe it.

# Addition

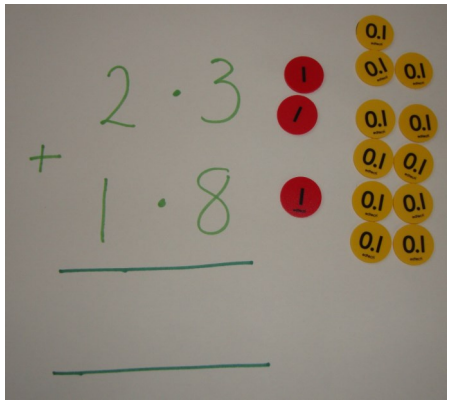
Year 5

- add and subtract whole numbers with more than 4 digits,
- including using formal written methods (columnar addition and subtraction)
- add and subtract numbers mentally with increasingly large numbers

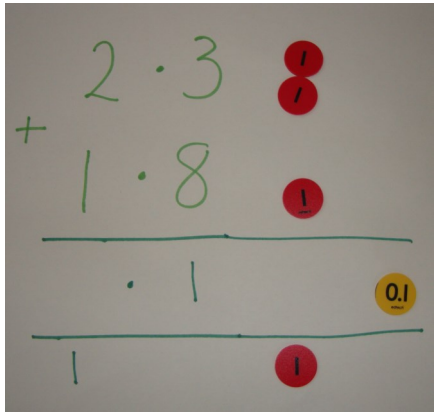
Non Statutory (Fractions)

- They mentally add and subtract tenths, and one digit whole numbers and tenths
- use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy

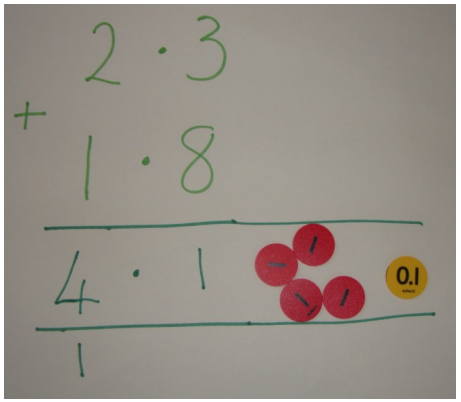
## Working with decimals




Add up tenths first then whole numbers.



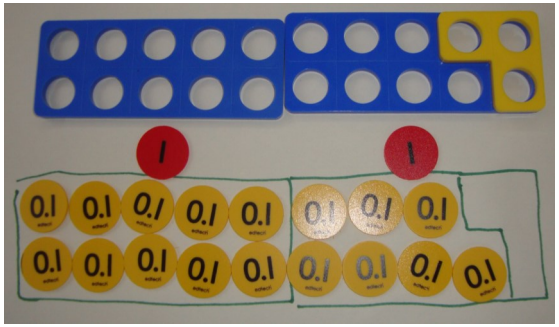

10 x 0.1-1 whole so exchange for 1 counter. 0.1 left in tenths column.




# Subtraction

Children should continue to use concrete apparatus as required. They need to understand the structure of the maths.

This includes for decimals



2 - 0.3 = 1.7

1.7 + ☐ = 2

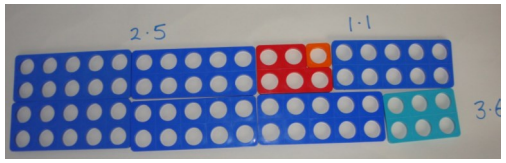
Year 6

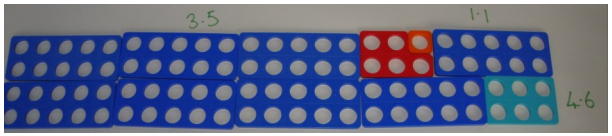
solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why

**True or false?**

Explain how you know *without* calculating.

3.6 - 2.5 = 4.6 - 3.5



# Statutory Guidance– Written Methods

## Addition and subtraction

789 + 642 becomes

$$\begin{array}{rrrr} & 7 & 8 & 9 \\ + & 6 & 4 & 2 \\ \hline 1 & 4 & 3 & 1 \\ \hline & {\scriptstyle 1} & {\scriptstyle 1} & \end{array}$$

Answer: 1431

874 – 523 becomes

$$\begin{array}{rrrr} & 8 & 7 & 4 \\ - & 5 & 2 & 3 \\ \hline & 3 & 5 & 1 \\ \hline \end{array}$$

Answer: 351

932 – 457 becomes

$$\begin{array}{rrrr} & {\scriptstyle 8} & {\scriptstyle 1\,2} & {\scriptstyle 1} \\ & \cancel{9} & \cancel{3} & 2 \\ - & 4 & 5 & 7 \\ \hline & 4 & 7 & 5 \\ \hline \end{array}$$

Answer: 475

932 – 457 becomes

$$\begin{array}{rrrr} & & {\scriptstyle 1} & {\scriptstyle 1} \\ & 9 & 3 & 2 \\ - & \cancel{4} & \cancel{5} & 7 \\ & {\scriptstyle 5} & {\scriptstyle 6} & \\ \hline & 4 & 7 & 5 \\ \hline \end{array}$$

Answer: 475

National Curriculum 2014– Appendices

See NCETM for useful Videos modelling how to use Concrete Resources to support written methods.

https://www.ncetm.org.uk/resources/40532 Developing Column Subtraction