Hampshire Services
HIAS SCHOOL IMPROVEMENT

MATHEMATICS ADVISORY CENTRE

Progression in Calculation Multiplication and Division

Hampshire County Council

www.hants.gov.uk

Version 1 - October 2016

How to use this document

This document is designed to give ideas about how to use concrete apparatus and images to support children's conceptual understanding.

So often children are able to follow calculation processes and get the answer right in many cases without fully understanding how and why the method works. This document is designed to support teachers to do that so that children are able to reason effectively and apply maths that they have learnt to other methods and new concepts as they have that deep understanding required at mastery. Not all National Curriculum objectives are listed but just some to give a flavour of how the images and apparatus can be used to develop children's conceptual understanding of different mathematical ideas. Teachers should read previous year groups to ensure children have consolidated these ideas before moving on. For some year groups the same images and concrete resources will be used but maybe with larger numbers and with a greater level of reasoning so they have therefore not been repeated.

For more guidance use the Resource Tool https://www.ncetm.org.uk/resources/41211 on the NCETM website where exemplification material and activity ideas can be found using the following tabs of EXEMPLIFICATION and ACTIVITIES. The SUBJECT KNOWLEDGE Audit tool will also help to give an insight to the expectations under each NC objective. There is also a tab for VIDEOS under each domain where you can see teacher using concrete resources to develop childrens' conceptual understanding and reasoning.

The EYFS Framework

Mathematics involves providing children with opportunities to develop and improve their skills in counting, understanding and using numbers, calculating simple addition and subtraction problems; and to describe shapes, spaces, and measures

Early Learning Goals

Mathematics Numbers: children count reliably with numbers from 1 to 20, place them in order and say which number is one more or one less than a given number. Using quantities and objects, they add and subtract two single-digit numbers and count on or back to find the answer. They solve problems, including doubling, halving and sharing.

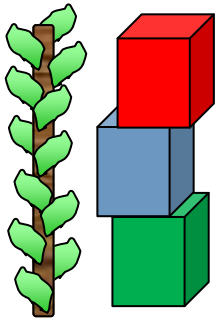
Shape, space and measures: children use everyday language to talk about size, weight, capacity, position, distance, time and money to compare quantities and objects and to solve problems. They recognise, create and describe patterns. They explore characteristics of everyday objects and shapes and use mathematical language to describe them.

Additional Guidance can be found in Development Matters.

https://www.early-education.org.uk/development-matters

Early Maths Foundation Stage to early KS1

ELG: Children count reliably with numbers from one to 20, place them in order and say which number is one more or one less than a given number. Using quantities and objects, they add and subtract two single-digit numbers and count on or back to find the answer. They solve problems, including doubling, halving and sharing.



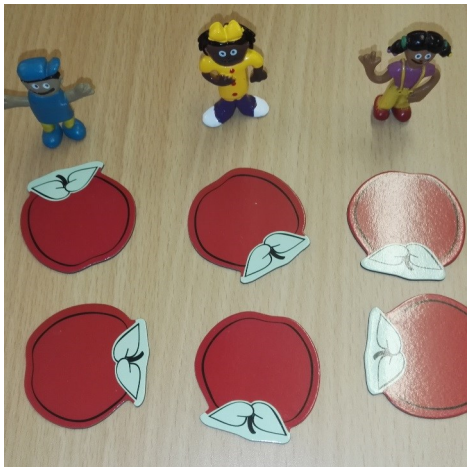
Each day Jack's beanstalk doubled in height. It was twice as tall.

Today it is 3 bricks tall. How tall will it be tomorrow?

ELG

They solve problems, including doubling, halving and sharing.

Cut the food in half to share with a friend.



3 friends wanted to share the last 6 apples. To make it fair they need the same amount each.

Key Stage 1

The principal focus of mathematics teaching in key stage 1 is to ensure that pupils develop confidence and mental fluency with whole numbers, counting and place value. This should involve working with numerals, words and the four operations, including with practical resources [for example, concrete objects and measuring tools].

At this stage, pupils should develop their ability to recognise, describe, draw, compare and sort different shapes and use the related vocabulary. Teaching should also involve using a range of measures to describe and compare different quantities such as length, mass, capacity/volume, time and money.

By the end of year 2, pupils should know the number bonds to 20 and be precise in using and understanding place value. An emphasis on practice at this early stage will aid fluency.

Pupils should read and spell mathematical vocabulary, at a level consistent with their increasing word reading and spelling knowledge at key stage 1.

National Curriculum 2014

Counting and Place Value

Year 1

Count in multiples of two, five and ten.

50p £1 (100p) £1.50 (150p) £2 (200p)

Year 2

Count in steps of 2, 3, 5 from 0 and in tens from any number forwards and backwards

Year 2

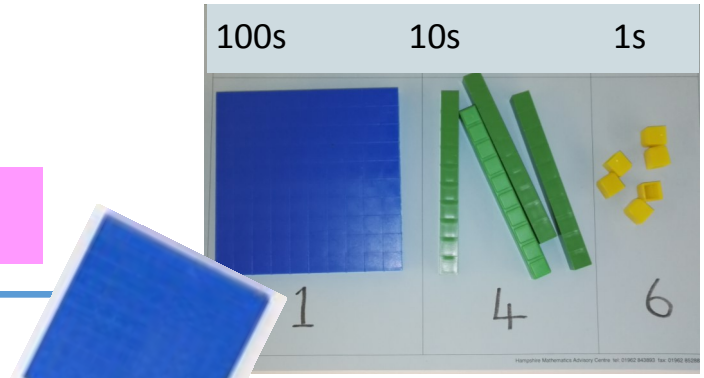
Recall and use multiplication tables and division facts for the 2, 5 and 10 multiplication tables.

3 x 5 = 15

How many 3s in 15?

How many groups of 3 in 15?

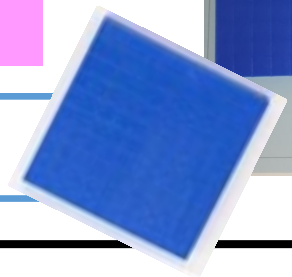
3 x □ = 15



Year 3

Count in zero from multiples of 4, 8, 50, 100 and find 100 more or less than a given num-

What will change/stay the same if I add/subtract another 100?



Using objects and pictorial representations alongside concrete resources

If I have 6 socks. How many pairs will that make?

3 pairs

Year 1

Solve one step problems involving multiplication and division by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.

Year 2

Solve problems using multiplication and division, using materials, arrays, repeated addition , mental methods and multiplication and division facts, including problems in contexts.

15 frogs ÷ 3 lilly pads = ☐ Frogs on each

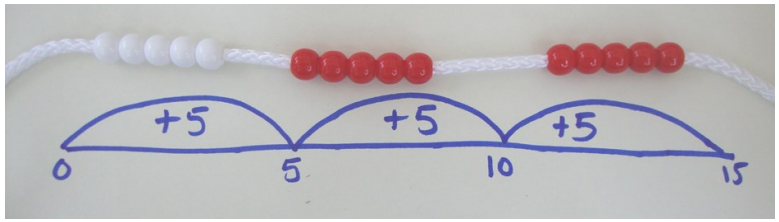
5 frogs on each lily pad

5 x 3 = 15

☐ frogs X 3 lilly pads = 15

There are 15 frogs. There are the same amount on each Lilly pad. If there are 3 Lilly pads, how many are sat on each one?

Repeated Addition

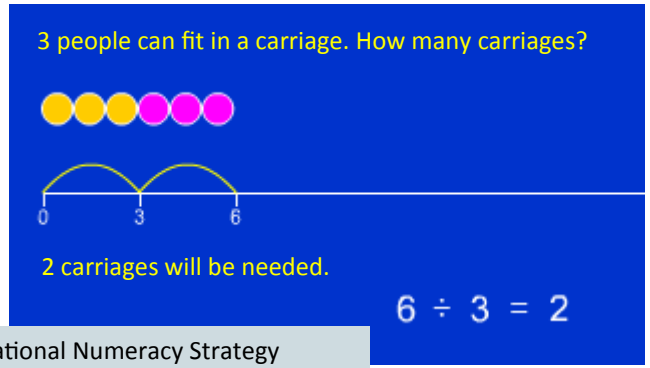


Division as grouping and sharing

3 people will fit in a carriage. How many carriages will I need to carry 6 people?

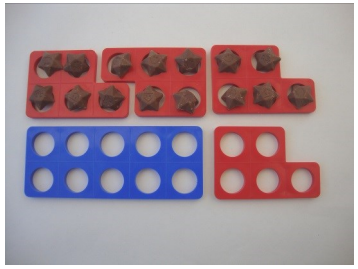
Year 1

Solve one step problems involving multiplication and division by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.



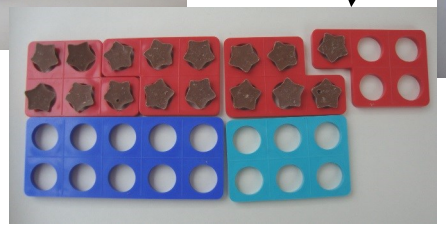
ITPs National Numeracy Strategy

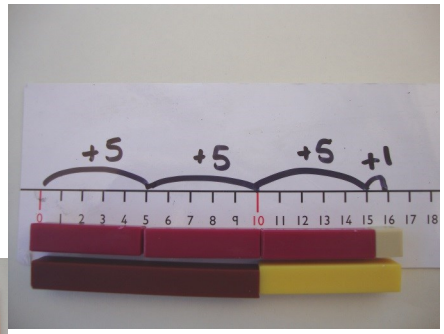
5 chocolates will fit in a box. How many boxes will I need for 15 chocolates?



What if I had 16 chocolates...how many boxes would I need then?

Extra box= 4 boxes!



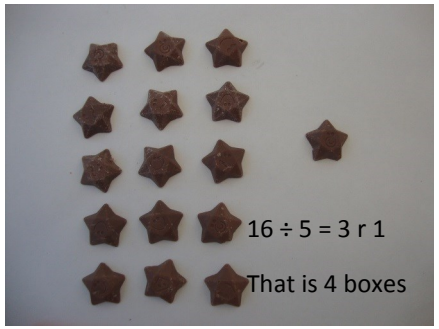


There are 6 fish. How many bowls will I need if I want 2 fish in each?

There are 3 bowls, each with 2 fish!

I will group in 2s or share my fish across the 3 bowls.

Which is more efficient?



Year 2

Solve problems using multiplication and division, using materials, arrays, repeated addition , mental methods and multiplication and division facts, including problems in contexts.

Multiplication- Repeated addition, arrays and multiples

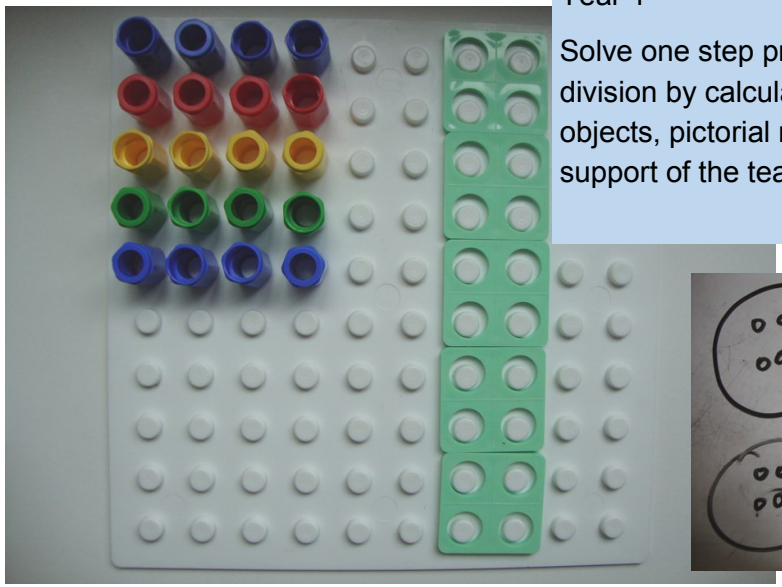
4 x 1

4 x 2

4 x 3

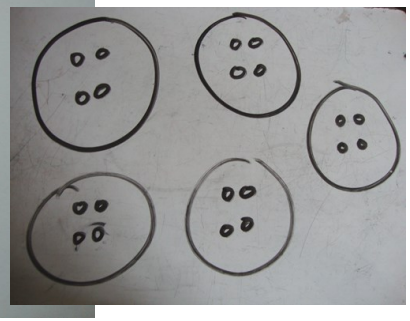
4 x 4

4 x 5



Year 1

Solve one step problems involving multiplication and division by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.



4 Cheerios in one bowl, how many in 5 bowls?

4 + 4 + 4 + 4 + 4 = 20

4 x 5 = 20

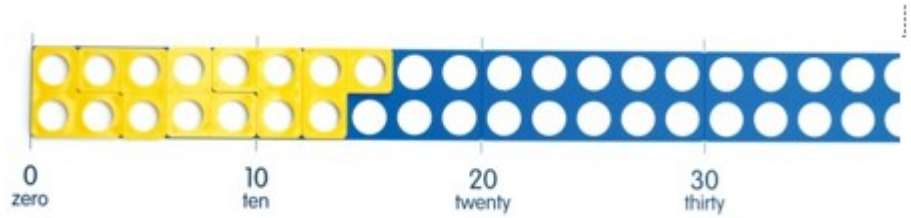
If 5 friends wanted to share 20 Cheerios, how many would they each have? 20 Cheerios ÷ 5 people = 4 Cheerios each

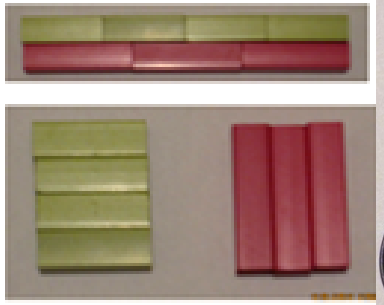
Year 2

Solve problems using multiplication and division, using materials, arrays, repeated addition , mental methods and multiplication and division facts, including problems in contexts.

3 + 3 + 3 + 3 + 3

5 x 3 = 15





4 x 3 = 3 x 4

How many pies?

Lower Key Stage 2

The principal focus of mathematics teaching in lower key stage 2 is to ensure that pupils become increasingly fluent with whole numbers and the four operations, including number facts and the concept of place value. This should ensure that pupils develop efficient written and mental methods and perform calculations accurately with increasingly large whole numbers.

At this stage, pupils should develop their ability to solve a range of problems, including with simple fractions and decimal place value. Teaching should also ensure that pupils draw with increasing accuracy and develop mathematical reasoning so they can analyse shapes and their properties, and confidently describe the relationships between them. It should ensure that they can use measuring instruments with accuracy and make connections between measure and number.

By the end of year 4, pupils should have memorised their multiplication tables up to and including the 12 multiplication table and show precision and fluency in their work.

Pupils should read and spell mathematical vocabulary correctly and confidently, using their growing word reading knowledge and their knowledge of spelling.

National Curriculum 2014

Counting and place value

Year 3
- Count in zero in multiples of 4, 8, 50, 100...

50p, £1, £1.50, £2

Or 50p, 100p, 150p, 200p

Count in different contexts using the language associated with the context.

Show number lines in many orientations and count in multiples.

50ml, 100ml, 150ml.

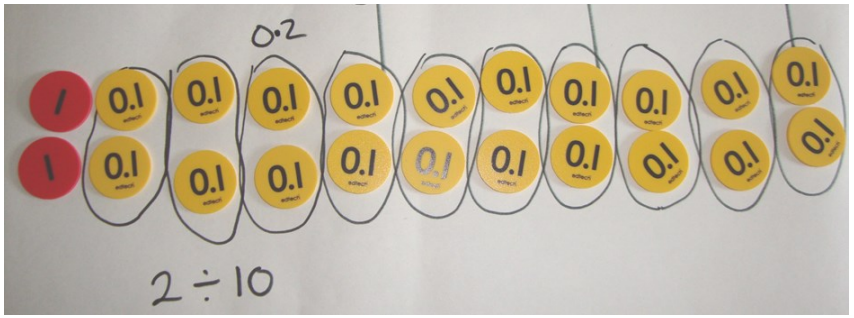
Ordering and comparing numbers

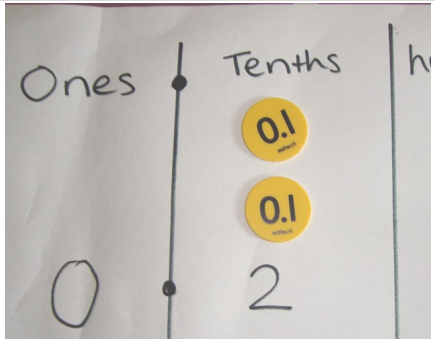
What numbers could lie between these two values?

Which number is halfway between… and …?

Year 3 (Fractions)
Count up and down in tenths; recognise that tenths arise from dividing an object into 10 equal parts and in dividing one digit numbers or quantities by 10.
- .

Year 4 (Fractions)
- Count up or down in hundredths; recognise hundredths arise when dividing an object by one hundred and dividing tenths by ten.





Developing Reasoning and Application to other domains

Complete:

6 x 2 =

6 x 10 =

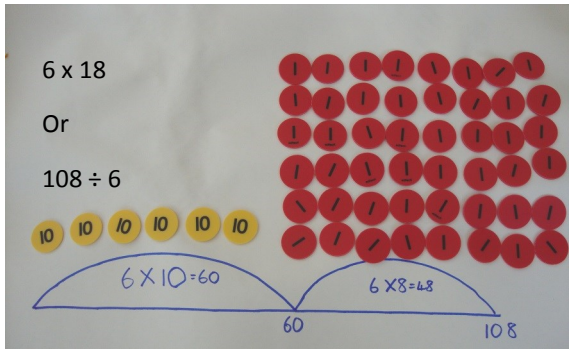
6 x 20 =

6 x 22 =

What is the same/different? Model these to show the connections

Year 3

- Write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit times one-digit numbers, using mental methods and progressing to formal written methods



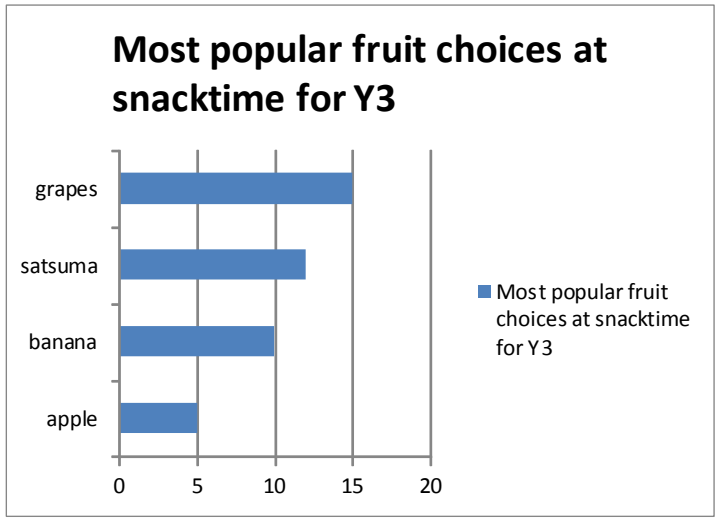
Children need a good grasp of using multiplication and division facts to allow them to use informal jottings to solve simple calculations mentally using recall of known facts.

Year 3 (Statistics)
Solve one and two step problems using information presented in scaled bar charts and pictograms
Non– Statutory
Pupils use simple scales e.g. 2, 5, 10 units per cm.

How many more people prefer cats to dogs?

5—2 = 3

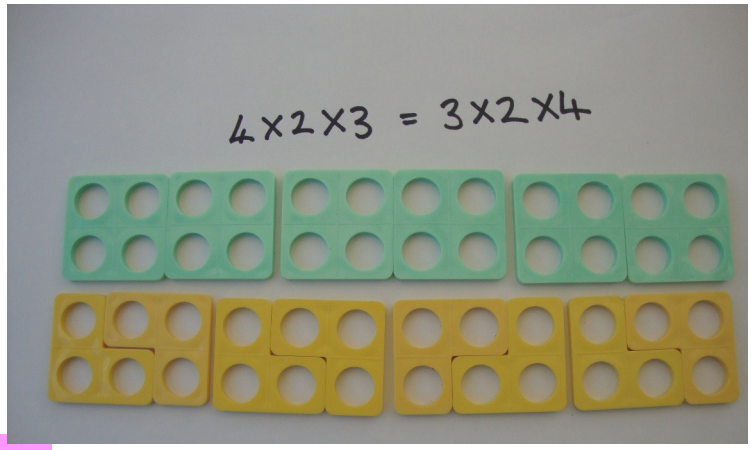
Favourite Pets	
Cat	🐾 🐾 🐾 🐾 🐾
Dog	🐾 🐾
Hamster	🐾 🐾 🐾
Each 🐾 stands for 2 votes	



How many more people preferred banana to apple?

How many people had a snack altogether?

Count in 5's to help you



4 x 2 x 3 = 3 x 2 x 4

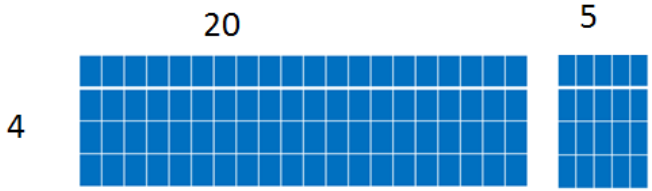
True or false? Prove it!

Year 3 **Non– Statutory**
Pupils develop efficient methods, for example, using commutativity and associativity.

Moving towards formal written methods of multiplication and division

Multiplication

Ref: ITP: multiplication facts



Linking arrays and grid method

2 5

X 4

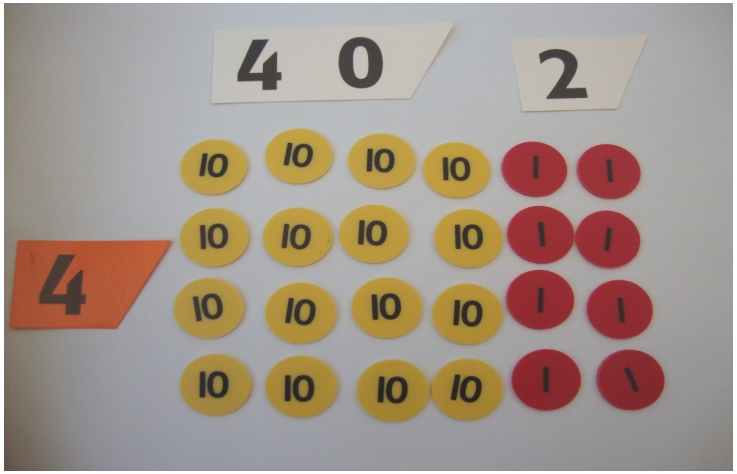
2 0 (5 x 4)

8 0 (20 x 4)

1 0 0

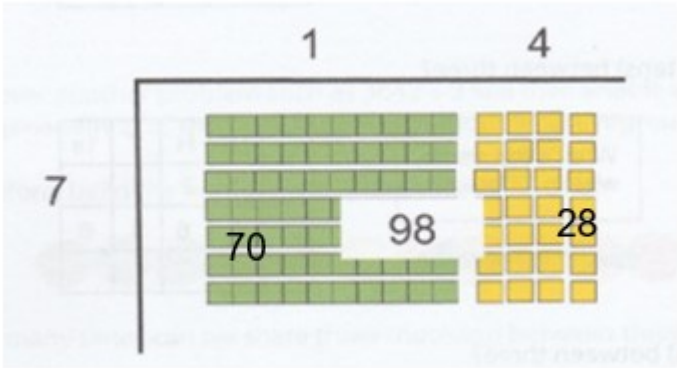
42 x 4 = 168

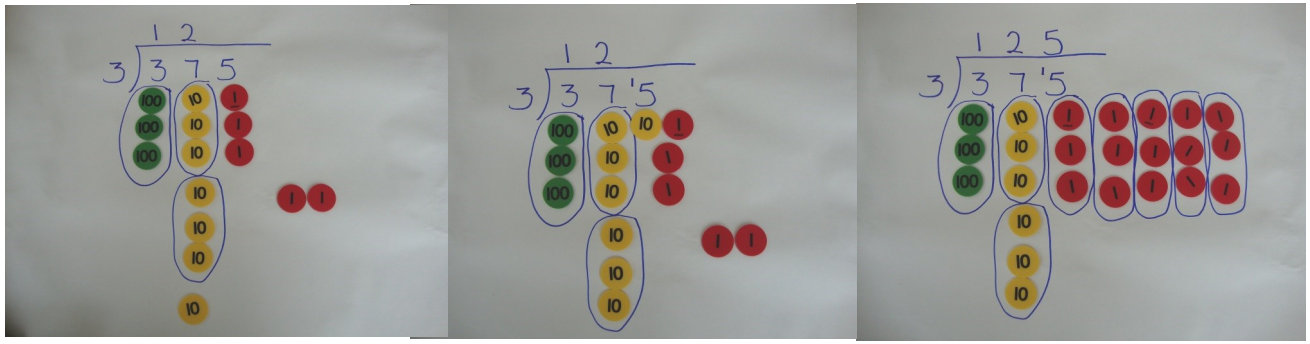
40 x 4 = 160



Division

Ref: NCETM





Exchange or regroup the ten that cannot be grouped into 3s.

For video model see: https://www.ncetm.org.uk/resources/43589

Year 4
Multiply 2 digit and 3 digit numbers by a one digit number using formal written layout. (see appendices of National Curriculum)

Upper Key Stage 2

The principal focus of mathematics teaching in upper key stage 2 is to ensure that pupils extend their understanding of the number system and place value to include larger integers. This should develop the connections that pupils make between multiplication and division with fractions, decimals, percentages and ratio.

At this stage, pupils should develop their ability to solve a wider range of problems, including increasingly complex properties of numbers and arithmetic, and problems demanding efficient written and mental methods of calculation. With this foundation in arithmetic, pupils are introduced to the language of algebra as a means for solving a variety of problems.

Teaching in geometry and measures should consolidate and extend knowledge developed in number. Teaching should also ensure that pupils classify shapes with increasingly complex geometric properties and that they learn the vocabulary they need to describe them.

By the end of year 6, pupils should be fluent in written methods for all four operations, including long multiplication and division, and in working with fractions, decimals and percentages.

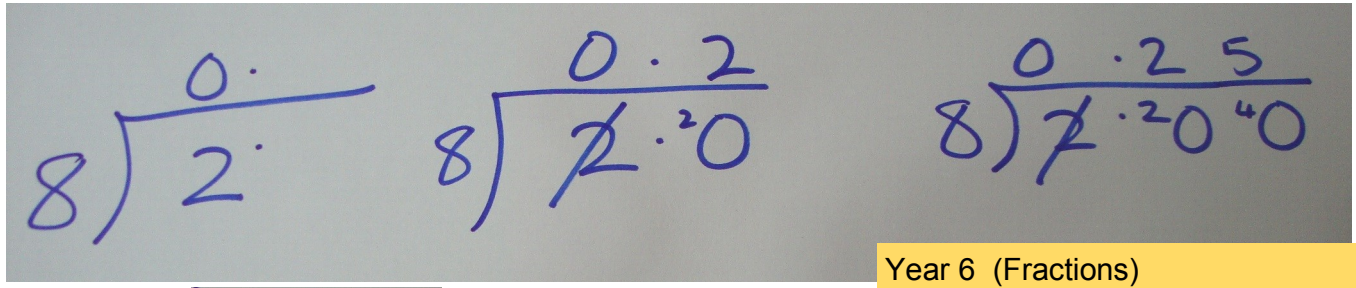
Pupils should read, spell and pronounce mathematical vocabulary correctly.

National Curriculum 2014

Developing written methods of multiplication and division

Division using decimals

Children need a good grasp of exchanging or regrouping where 0.1 x 10 = 1 and 0.01 x 10 = 0.1

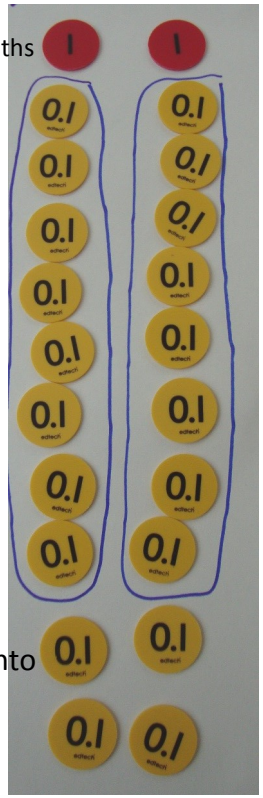


Group into 8s (as the divisor)

Year 6 (Fractions)

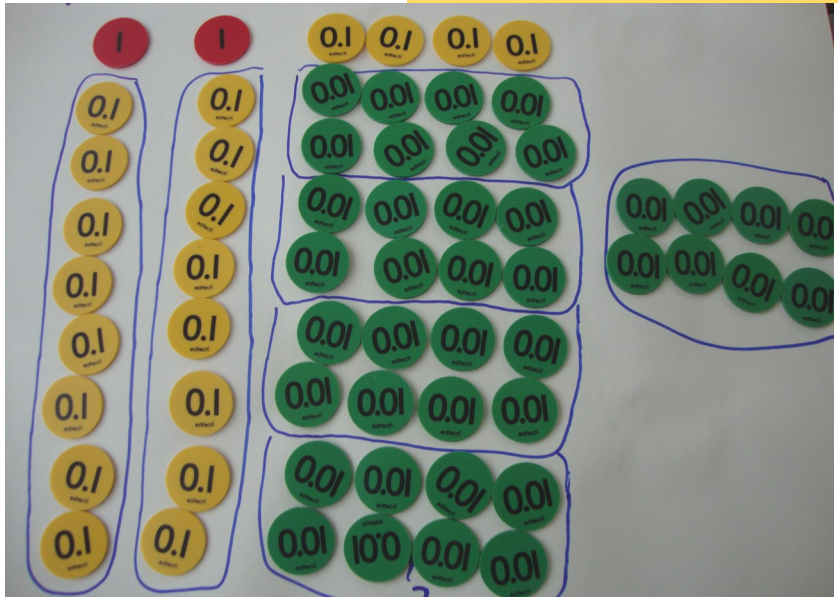
Use written division methods in cases where the answer has up to two decimal places.

regroup 2 into tenths



Group into 8

0.4 to regroup into hundredths



2 x 0.8 = 1.6 with 0.4 to regroup into hundredths

True or false? Prove it.

How do you know?

Show using a model.

0.2 x 5 = 0.5 x 2

0.2 x 4 + 0.2 = 0.2 x 5

Ref: Primary Magazine

Statutory Guidance– Formal Written Methods

Short multiplication

24 × 6 becomes

```
      2 4
  ×     6
  -------
  1 4 4
  -------
    2
```

Answer: 144

342 × 7 becomes

```
      3 4 2
  ×       7
  ---------
  2 3 9 4
  ---------
    2 1
```

Answer: 2394

2741 × 6 becomes

```
      2 7 4 1
  ×         6
  -----------
  1 6 4 4 6
  -----------
    4 2
```

Answer: 16 446

Long multiplication

24 × 16 becomes

```
      2
      2 4
  ×   1 6
  -------
  2 4 0
  1 4 4
  -------
  3 8 4
  -------
```

Answer: 384

124 × 26 becomes

```
      1 2
      1 2 4
  ×     2 6
  -----------
  2 4 8 0
    7 4 4
  -----------
  3 2 2 4
  -----------
  1 1
```

Answer: 3224

124 × 26 becomes

```
      1 2
      1 2 4
  ×     2 6
  -----------
    7 4 4
  2 4 8 0
  -----------
  3 2 2 4
  -----------
  1 1
```

Answer: 3224

National Curriculum 2014– Appendices

Statutory Guidance– Formal Written Methods

Short division

98 ÷ 7 becomes

```
      1  4
    ------
  7 | 9  8
      2
```

Answer: 14

432 ÷ 5 becomes

```
        8  6  r 2
    ---------
  5 | 4  3  2
      3
```

Answer: 86 remainder 2

496 ÷ 11 becomes

```
          4  5  r 1
      ------------
  1  1 | 4  9  6
          5
```

Answer: $45\frac{1}{11}$

Long division

432 ÷ 15 becomes

```
          2  8  r 12
  1  5 | 4  3  2
        3  0  0
        ---------
        1  3  2
        1  2  0
        ---------
           1  2
```

Answer: 28 remainder 12

432 ÷ 15 becomes

```
          2  8
  1  5 | 4  3  2
        3  0  0   15×20
        ---------
        1  3  2
        1  2  0   15×8
        ---------
           1  2
```

$\frac{\cancel{12}}{\cancel{15}} = \frac{4}{5}$

Answer: $28\frac{4}{5}$

432 ÷ 15 becomes

```
          2  8 · 8
  1  5 | 4  3  2 · 0
        3  0
        ---------
        1  3  2
        1  2  0
        ---------
           1  2  0
           1  2  0
           ---------
                 0
```

Answer: 28·8

National Curriculum 2014– Appendices

Moving to Formal Written Methods

Using the CPA approach

https://www.ncetm.org.uk/resources/40530 Multiple representations of Multiplication

https://www.ncetm.org.uk/resources/40530 Moving from grid to Column method.

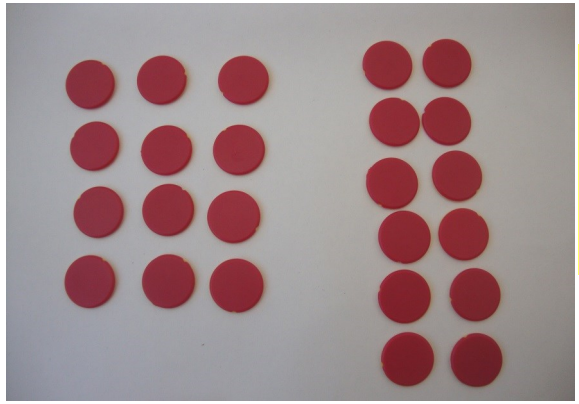
https://www.ncetm.org.uk/resources/43589, Moving to a written algorithm for division, Using Place Value Counters and recording division, Division with remainders, Division with exchange.

NCETM– Videos

Factors, Primes, Square and Cube Numbers and application to other domains

Use the counters to find factors of 12 by making arrays. What others can you find?
How many arrays can you make with 13 counters?

See Fractions, decimals, percentages and ratio and proportion booklet also.



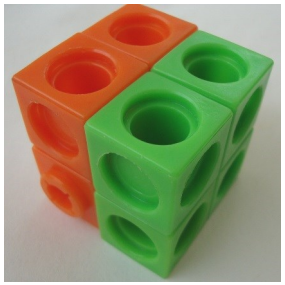
Year 5

Identify multiples and factors, including finding all factor pairs of a number and common factors of 2 numbers.

Now compare finding the factors of 15.

2 x 2 = 4

2 x 2 x 2 = 8



Year 4
Recognise and use factor pairs in mental calculation.

Year 5

Recognise and use square numbers and cube numbers, and use the notation for squared $^{(2)}$ and cubed $^{(3)}$.

The volume of the box is 150cm^{3}.

What could the dimensions be? Investigate.

☐ X ☐ X ☐ = 64

The number in the blue box is the same.

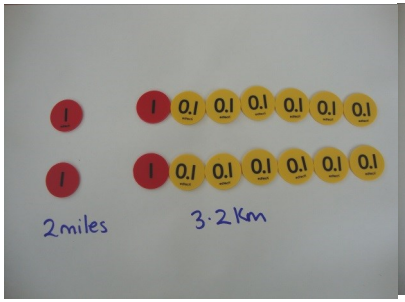
What could it be?

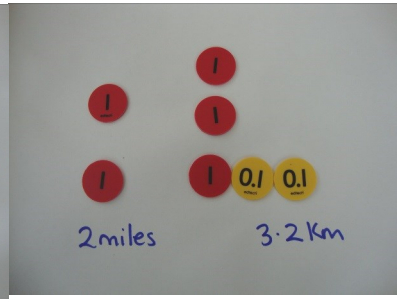
Year 6 Ratio and Proportion

Solve problems using the relative sizes of two quantities where missing values can be found by using integer multiplication and division facts.

1 mile = 1.6km

So 2 miles = 3.2km





Year 6 (Measures) *See Year 5 objectives also*

Convert between miles and km.

Solve problems involving the calculation and conversion of units of measure, using decimals notation up to 3 decimal places where appropriate.